Multiscale models for ocean–atmosphere exchanges

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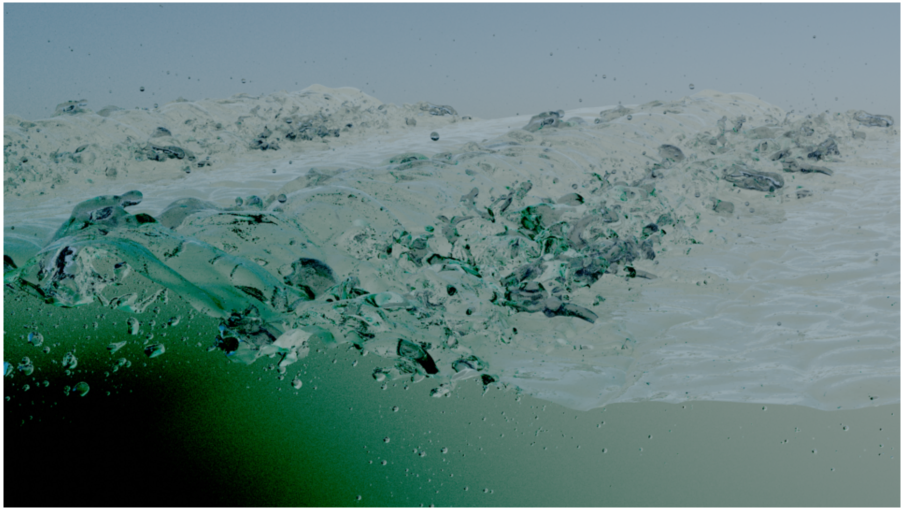
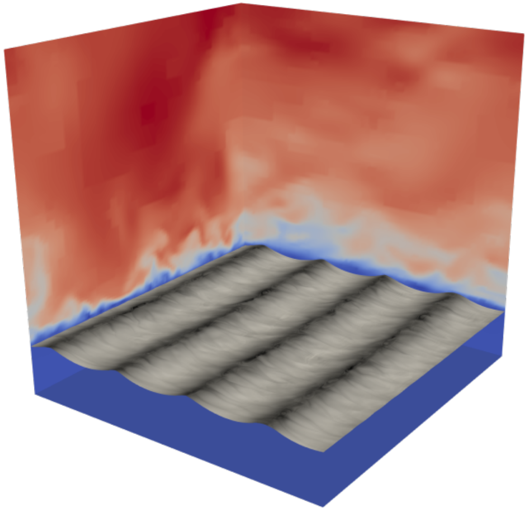
Global/Climate scale fluxes of momentum, heat, mass (water vapour, dissolved gases, areosols etc.) depend on a range of processes down to the microscale

Isabelle Gouttevin ce matin : *"Quelques lois physiques et beaucoup d'empirisme"*

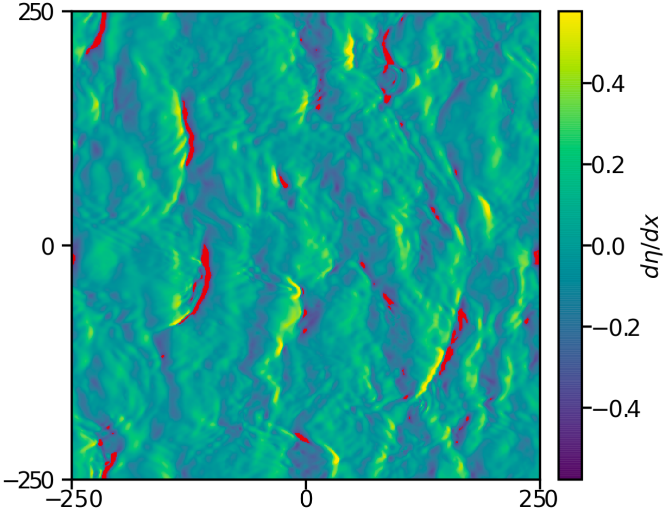
Graphical outline



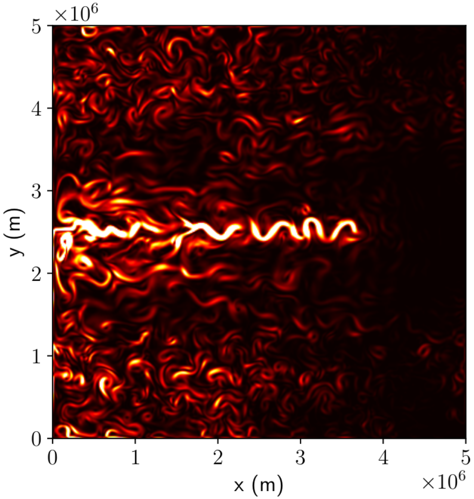
1 metre (Mostert et al, *JFM*, 2022)



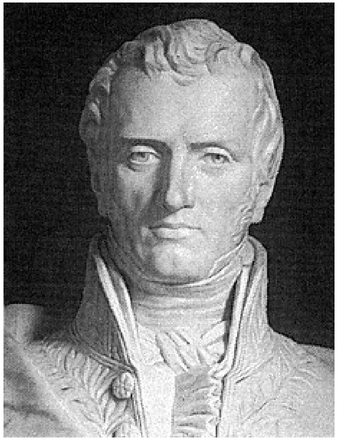
10 metres (Wu et al, *JFM*, 2022)



1 km (Wu et al, *JFM*, 2023)



1000 km (Uchida et al, *JPO*, 2022)



Claude-Louis Navier
1822

George Gabriel Stokes
1845

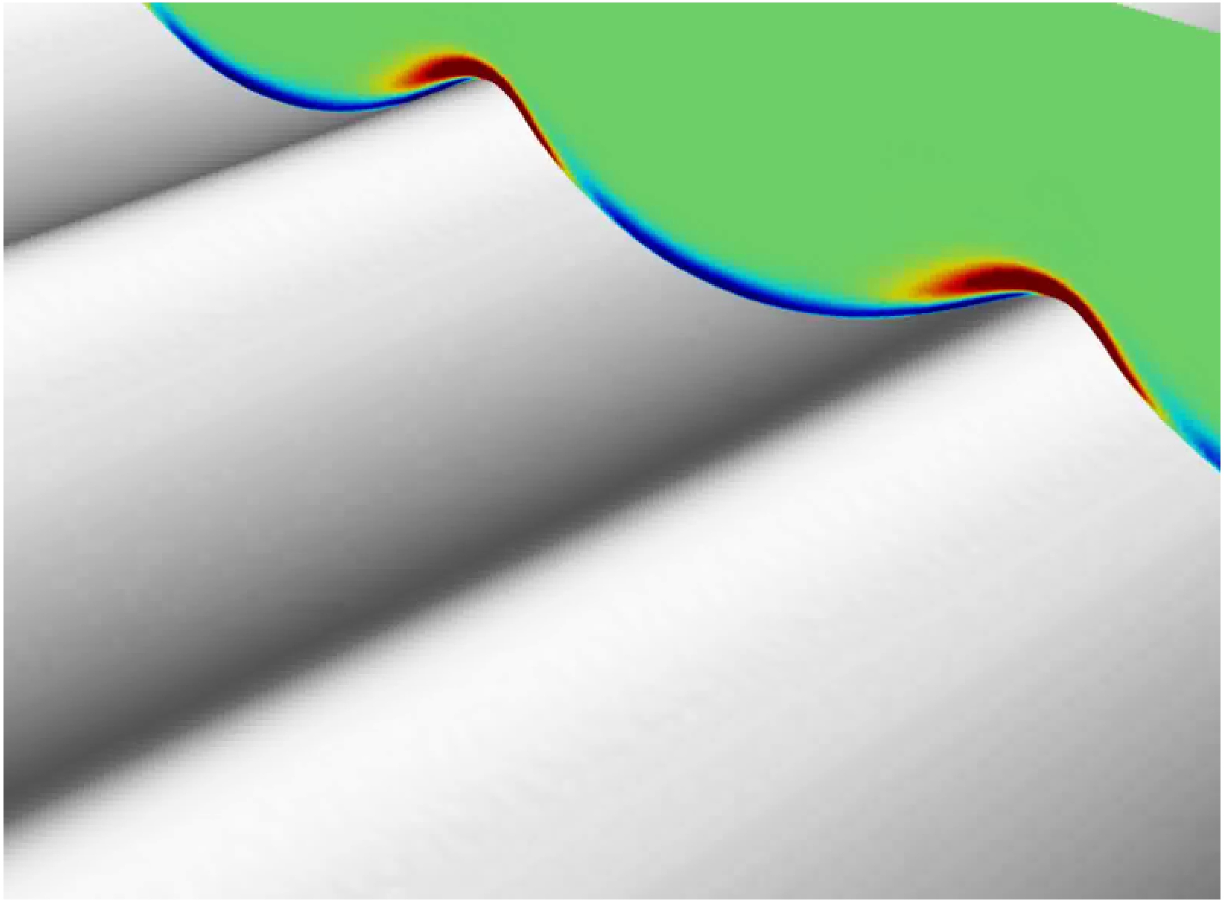
Adhémar de Saint-Venant
1819? 1843

Incompressible, variable-density and viscosity Navier–Stokes equations

$$
\begin{aligned}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) &= -\nabla p + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla^T \mathbf{u})] + \boldsymbol{S} \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}
$$

Source terms $\boldsymbol{S}$: gravity, surface tension, Coriolis etc.

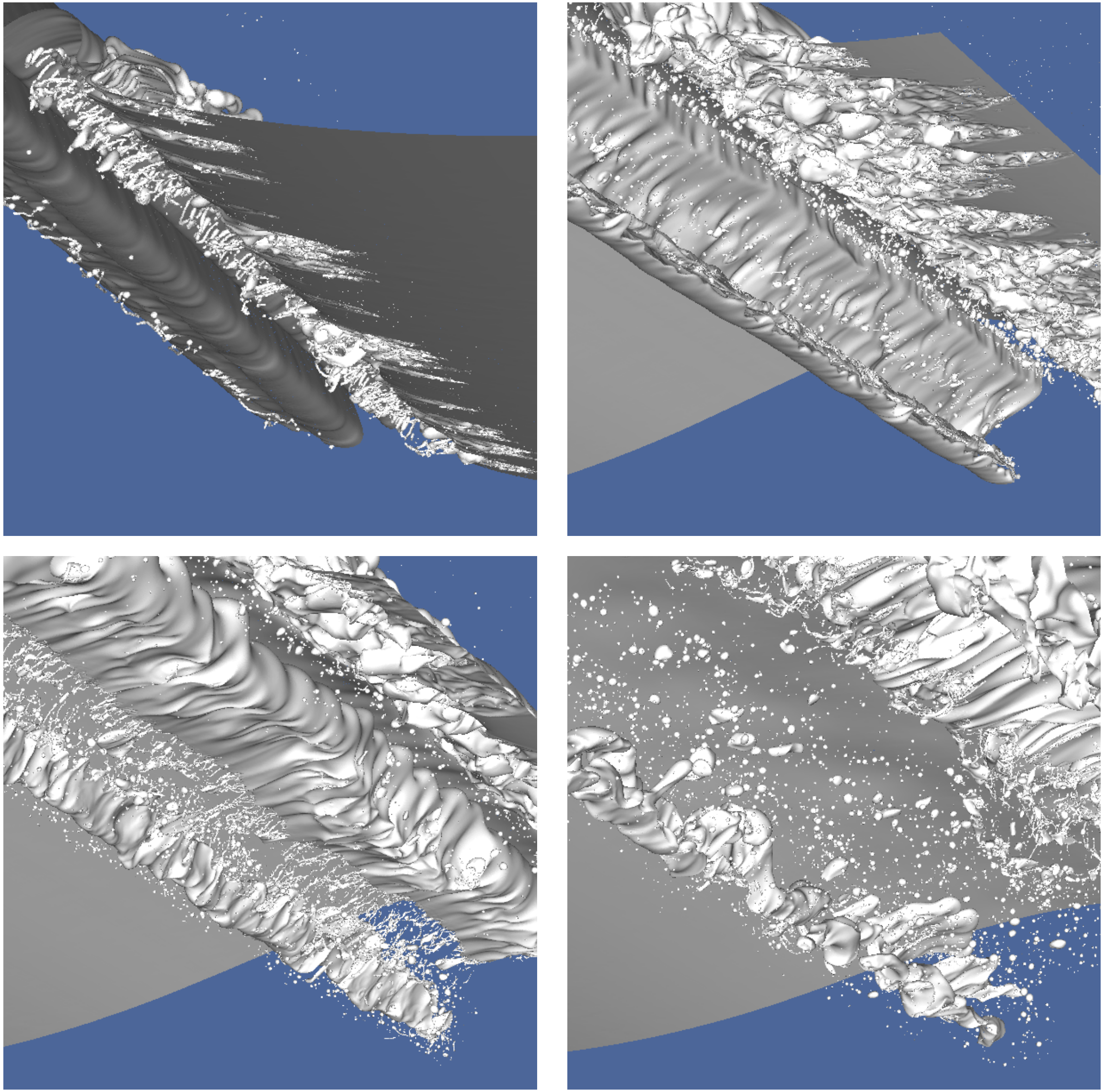
Important advances in their numerical approximation in the past 25 years



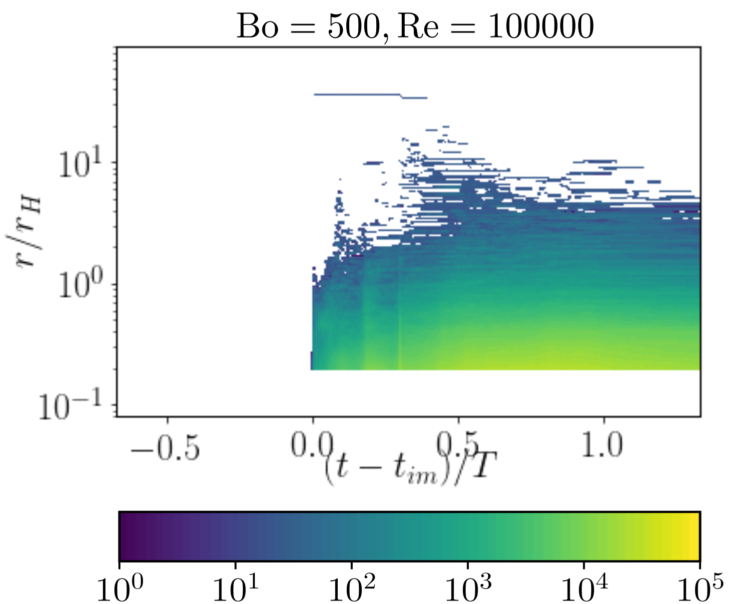
$\mathrm{Re} = 10^5$, $\mathrm{Bo} = 500$, $a\,k = 0.55$

2048^3 with adaptive mesh refinement, `basilisk.fr`

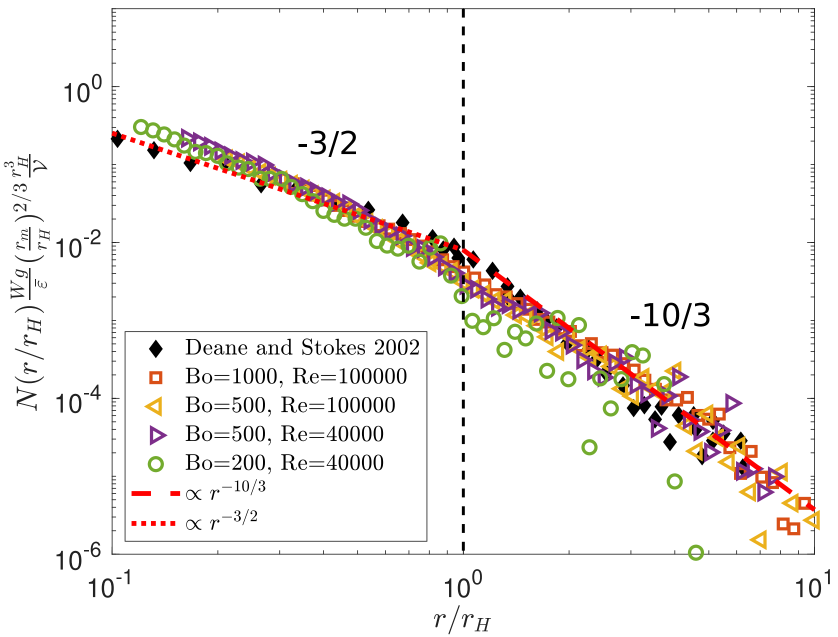
Underwater : a bubble breakup cascade



Bubble size distributions



Time-dependent



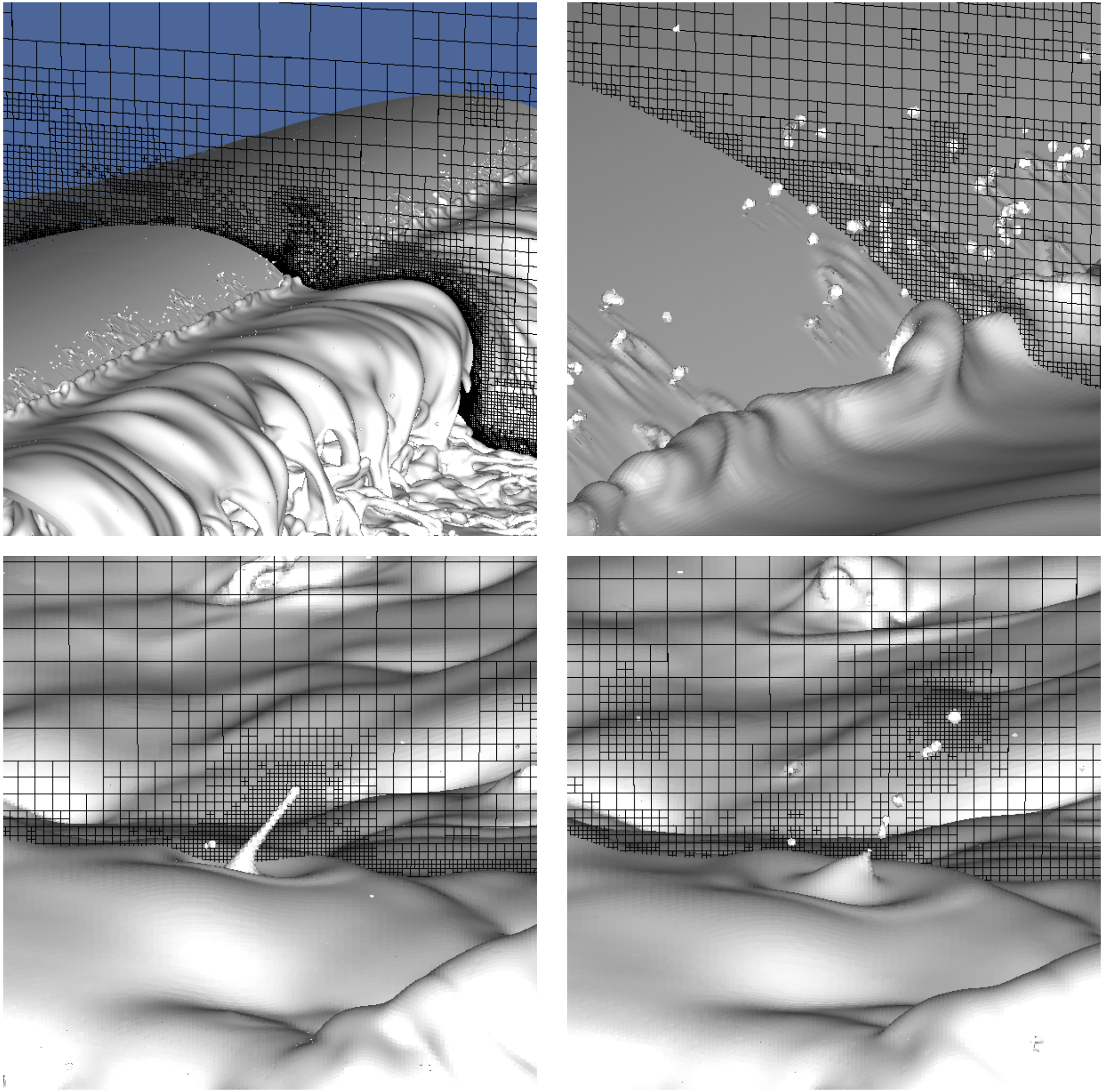
Time-averaged

Two distinct regimes described by a simple scaling relation:

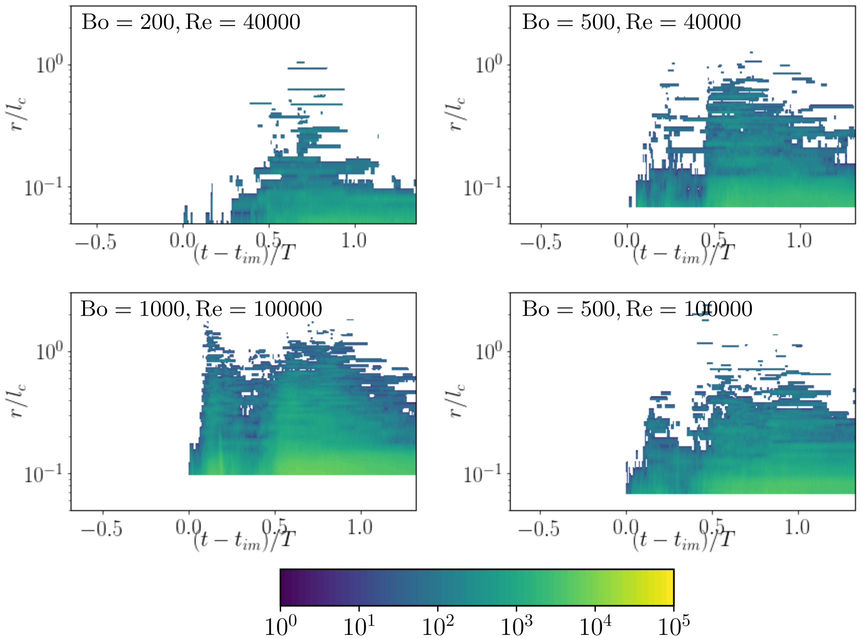
$$N(r/r_H) \propto (r/r_H)^{\alpha} \quad \text{with} \quad \alpha = -10/3 \quad \text{or} \quad \alpha = -3/2$$

The prefactor (but not the exponent) depends on the breaking-wave parameters

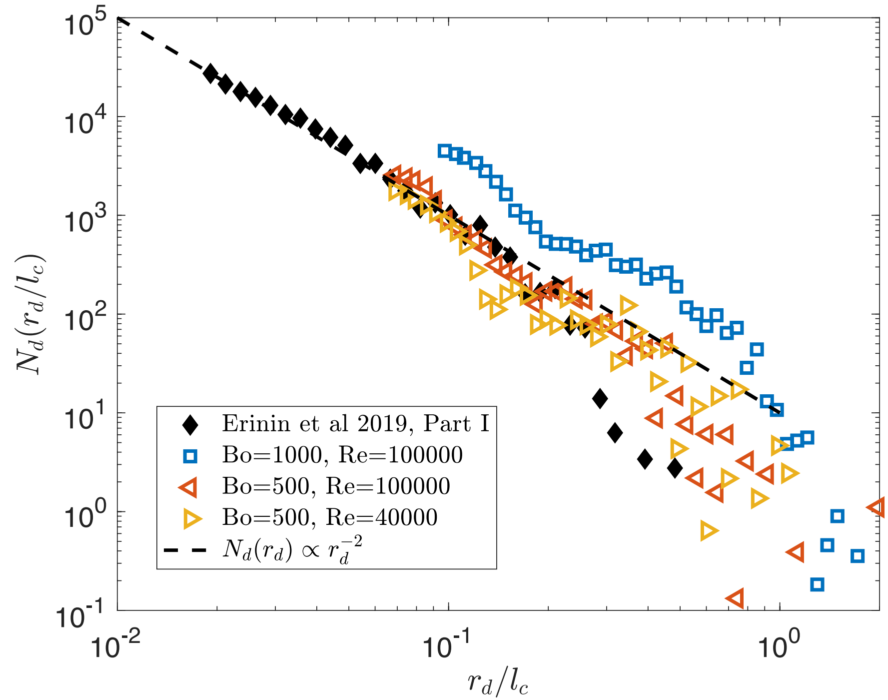
Methanation: $CO_2 + H_2$ + Energy $\longrightarrow$ Methane (Rolls-Royce MethanQuest)



Droplet size distributions



Time-dependent



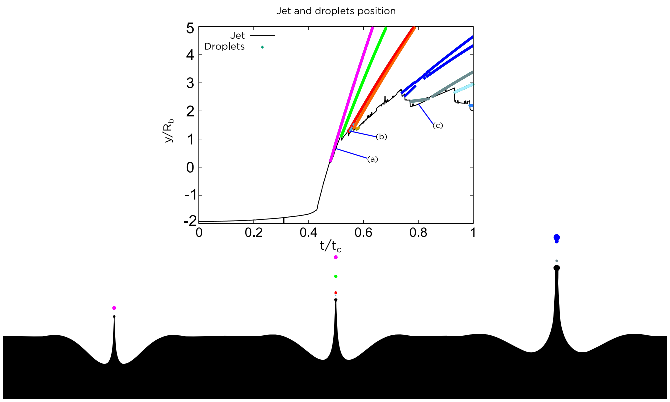
Time-averaged

Data still limited by computational cost / experimental difficulties

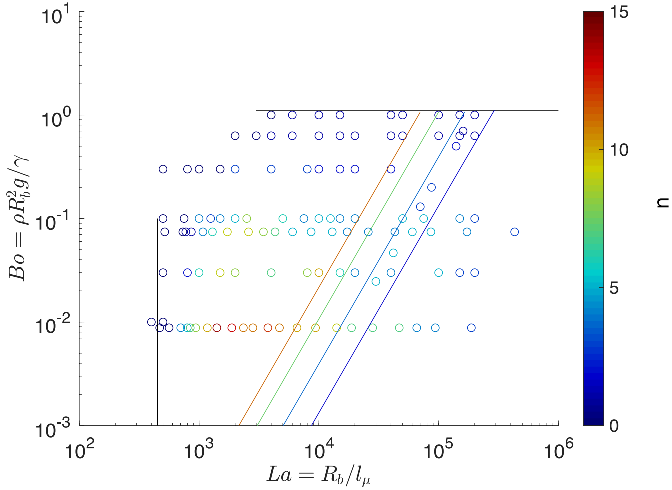
$\Rightarrow$ need a more detailed study of the generation mechanisms

Ghabache et al., 2016

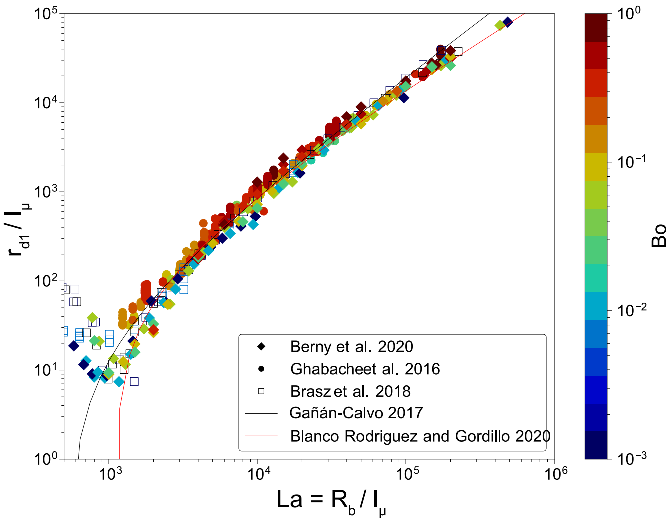
Generation of multiple droplets



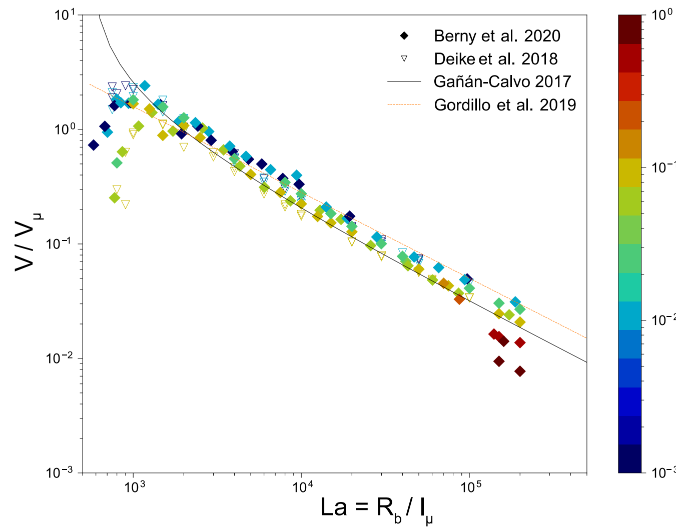
Number of droplets (La,Bo)



Droplet radius (La,Bo)



Ejection velocity (La,Bo)

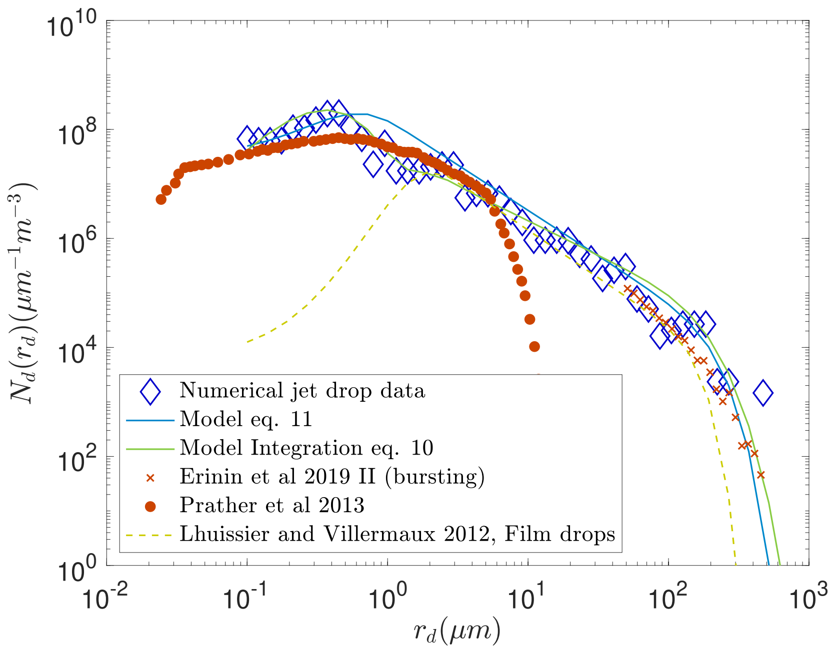


(Assume that) jet droplet production is dominated by sub-Hinze scale (breaking wave) bubbles i.e.

$$q(R_b) \;\propto\; R_b^{-3/2}$$

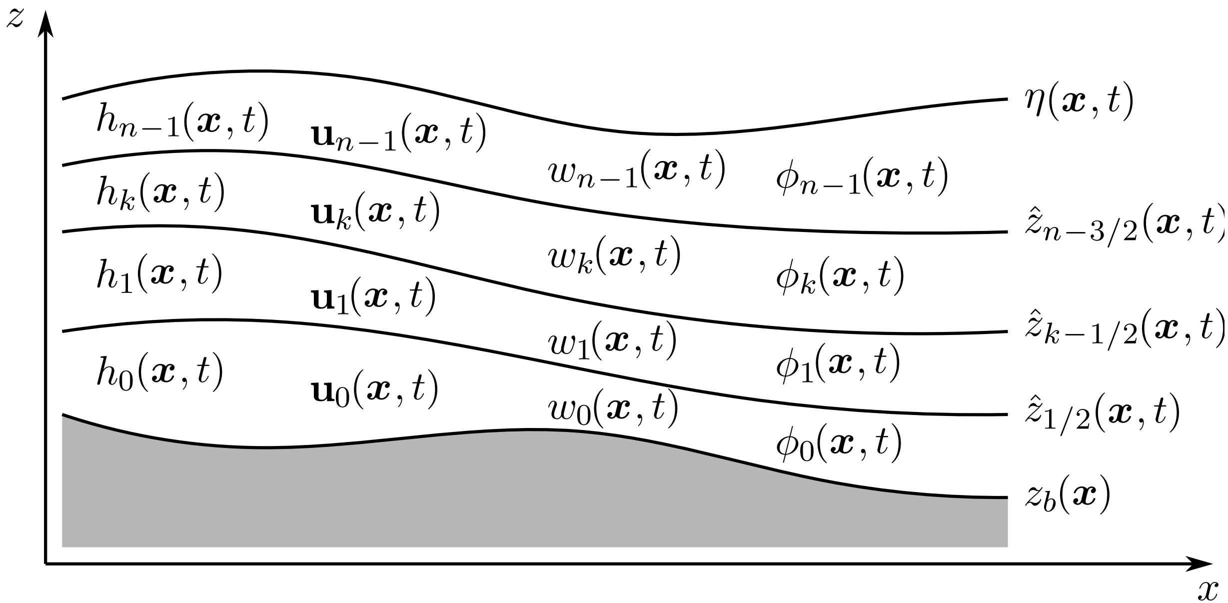
Convolution with the jet droplet distribution generated by a single bubble

$$N_d(r_d) \;=\; \int_{20\,\mu m}^{2.7mm} \frac{q(R_b)\,n(R_b)}{<r_d>}\, p(r_d/<r_d>, R_b)\, dR_b$$



The anisotropy at geophysical scales requires a different numerical method

"Multilayer" Lagrangian vertical description (Popinet, *JCP*, 2020)

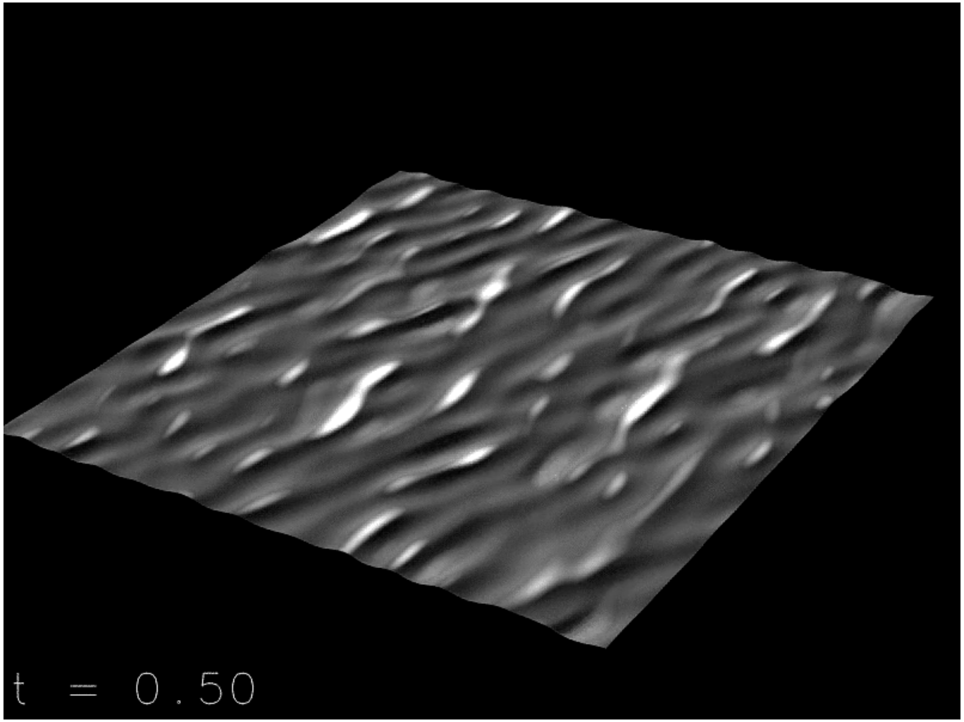


$$
\begin{aligned}
\partial_t h_k + \boldsymbol{\nabla}\cdot(h\,\mathbf{u})_k &= 0,\\
\partial_t (h\,\mathbf{u})_k + \boldsymbol{\nabla}\cdot(h\,\mathbf{u}\,\mathbf{u})_k &= -g\,h_k\,\boldsymbol{\nabla}\eta - \boldsymbol{\nabla}(h\,\phi)_k + [\phi\,\boldsymbol{\nabla} z]_k,\\
\partial_t (h\,w)_k + \boldsymbol{\nabla}\cdot(h\,w\,\mathbf{u})_k &= -[\phi]_k,\\
\boldsymbol{\nabla}\cdot(h\,\mathbf{u})_k + [w - \mathbf{u}\cdot\boldsymbol{\nabla} z]_k &= 0,\\
[f]_k &= f_{k+1/2} - f_{k-1/2}
\end{aligned}
$$

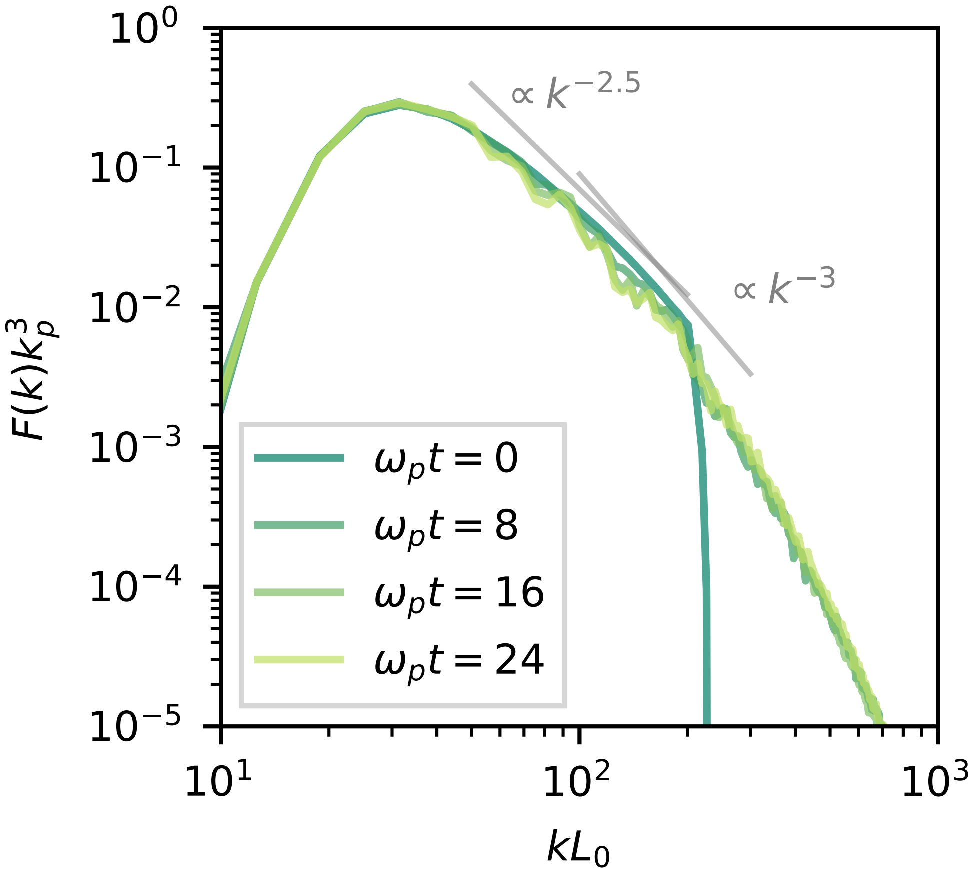
$$F(k,\theta) = Pk^{-5/2}\exp\left(-1.25\left(\frac{k_p}{k}\right)^2\right)\cos^N\theta$$

Pierson-Moskowitz (1964), JONSWAP Hasselmann et al. (1973)

No wind forcing (low dissipation)

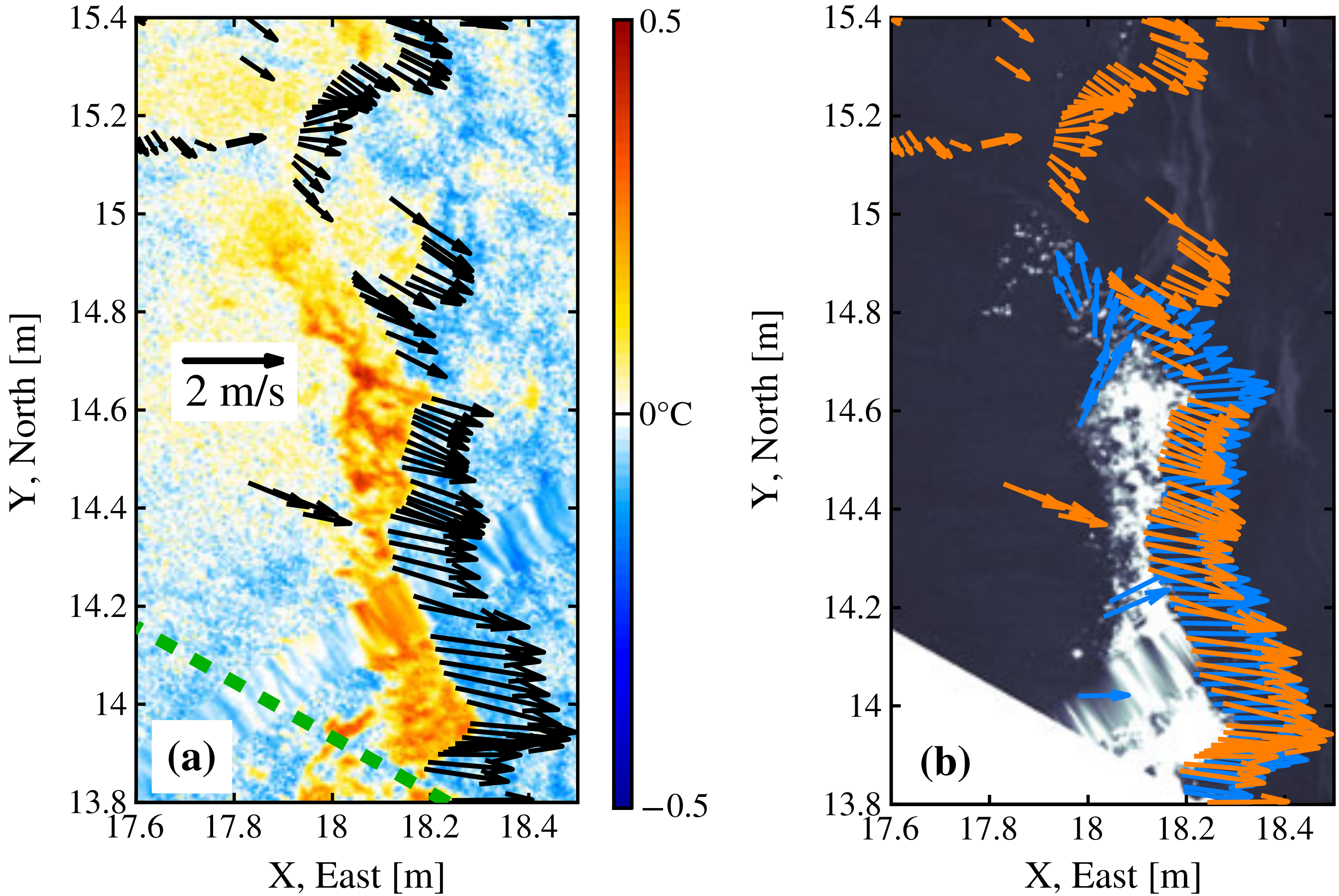


512^2, 50 layers, 16 initial modes, runtime a few hours on 64 cores

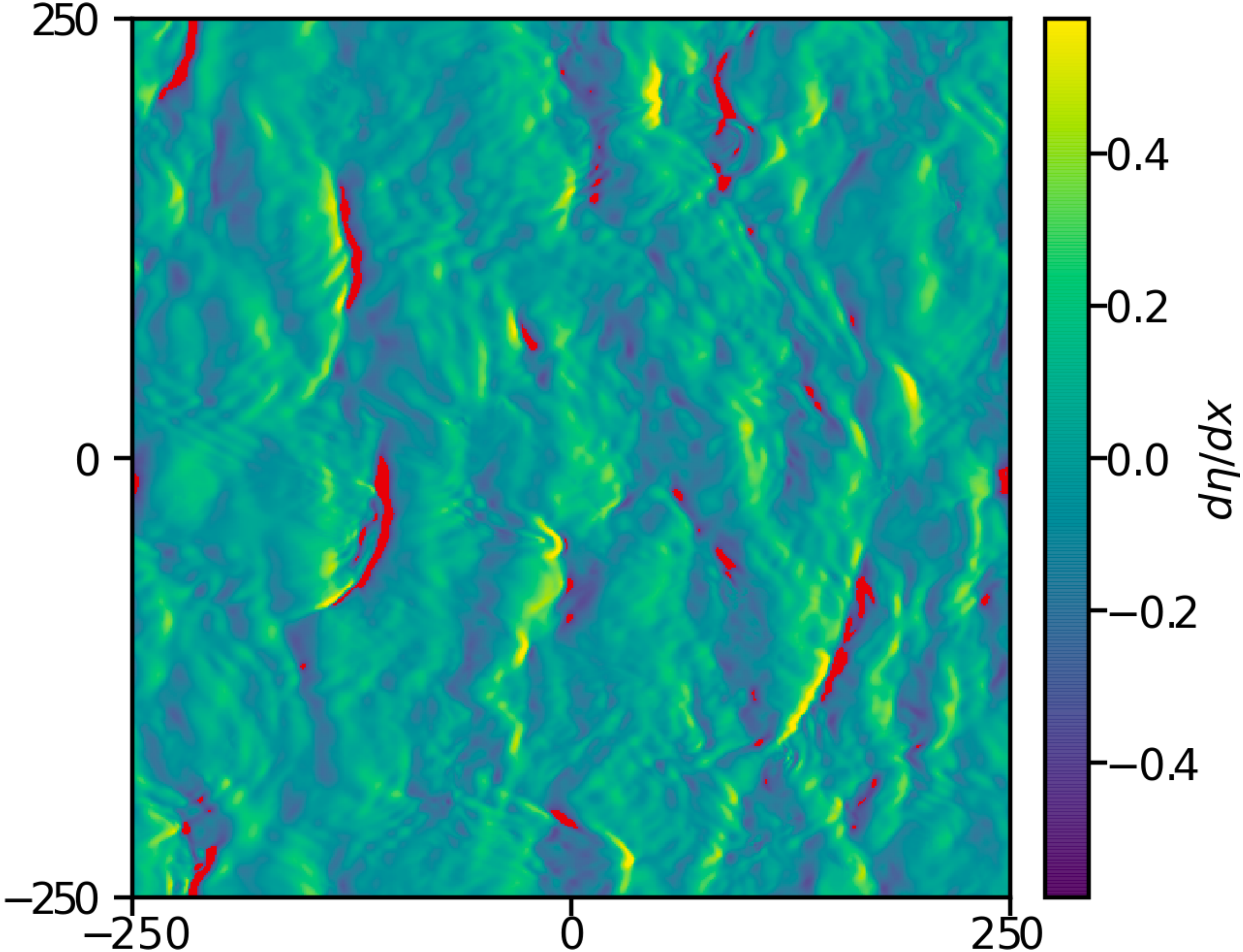


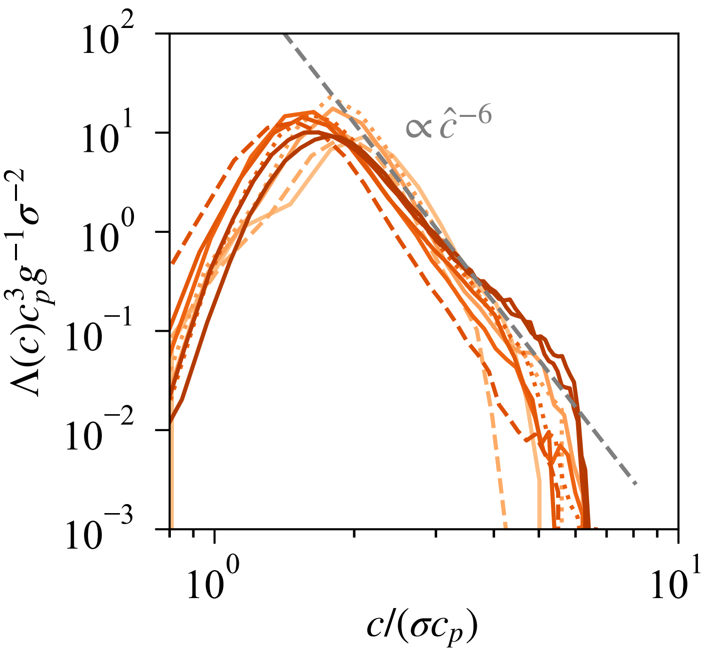
Convergence toward a realistic "equilibrium" spectrum i.e.
$F(k) \propto k^{-5/2} \exp\left(-1.25 \left(\frac{k_p}{k}\right)^2\right)$ + dissipative "roll-off"

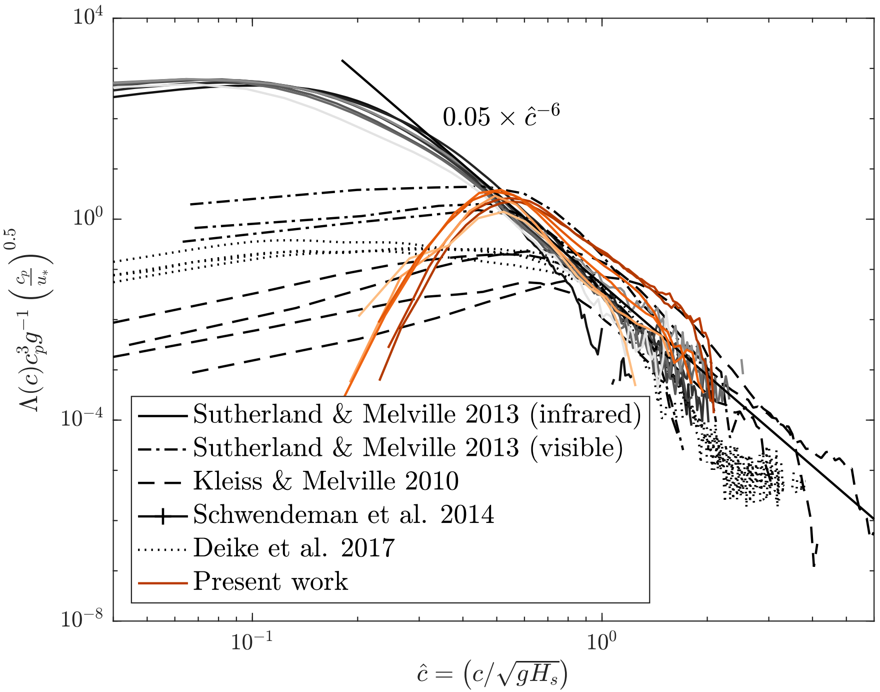
R/P FLIP (Scripps Oceanography), launched 1962



P. Sutherland and W. K. Melville (2013), Field measurements and scaling of ocean surface wave-breaking statistics, *Geophys. Res. Lett.*, 40, 3074–3079


250
0
−250
−250
0
250
0.4
0.2
0.0
−0.2
−0.4
$d\eta/dx$





Comparison with field data (Sutherland & Melville, GRL, 2013)

A simple semi-empirical relation to predict wave-breaking distributions

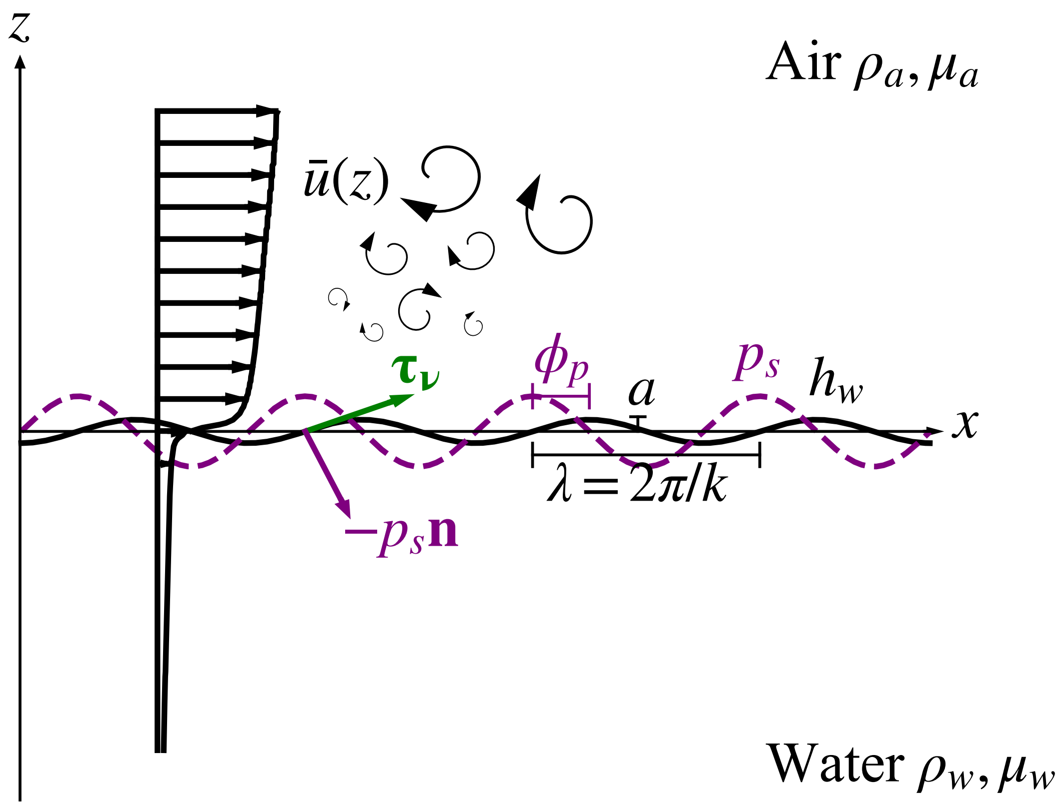
$$\Lambda(c)\, c_p^3\, g^{-1}\, (c_p/u_\star)^{1/2} \approx 0.05 \times \hat{c}^{-6}$$

Navier-Stokes with a free surface, Coriolis, temperature and salinity

$$
\begin{aligned}
\partial_t \mathbf{u} + \boldsymbol{\nabla} \cdot (\mathbf{u} \otimes \mathbf{u}) &= \frac{1}{\rho}(-\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) + \mathbf{g} + \boldsymbol{B}\,\mathbf{u} + \boldsymbol{\tau} \\
\boldsymbol{B} &= \begin{pmatrix} 0 & f \\ -f & 0 \end{pmatrix} \\
\boldsymbol{\nabla} \cdot \mathbf{u} &= 0 \\
\partial_t \boldsymbol{\chi}(\boldsymbol{x}, t) &= \mathbf{u}(\boldsymbol{\chi}(\boldsymbol{x}, t), t) \\
\partial_t T + \boldsymbol{\nabla} \cdot (\mathbf{u}\, T) &= \phi_T \\
\partial_t S + \boldsymbol{\nabla} \cdot (\mathbf{u}\, S) &= \phi_S \\
\rho &= \rho(S, T)
\end{aligned}
$$

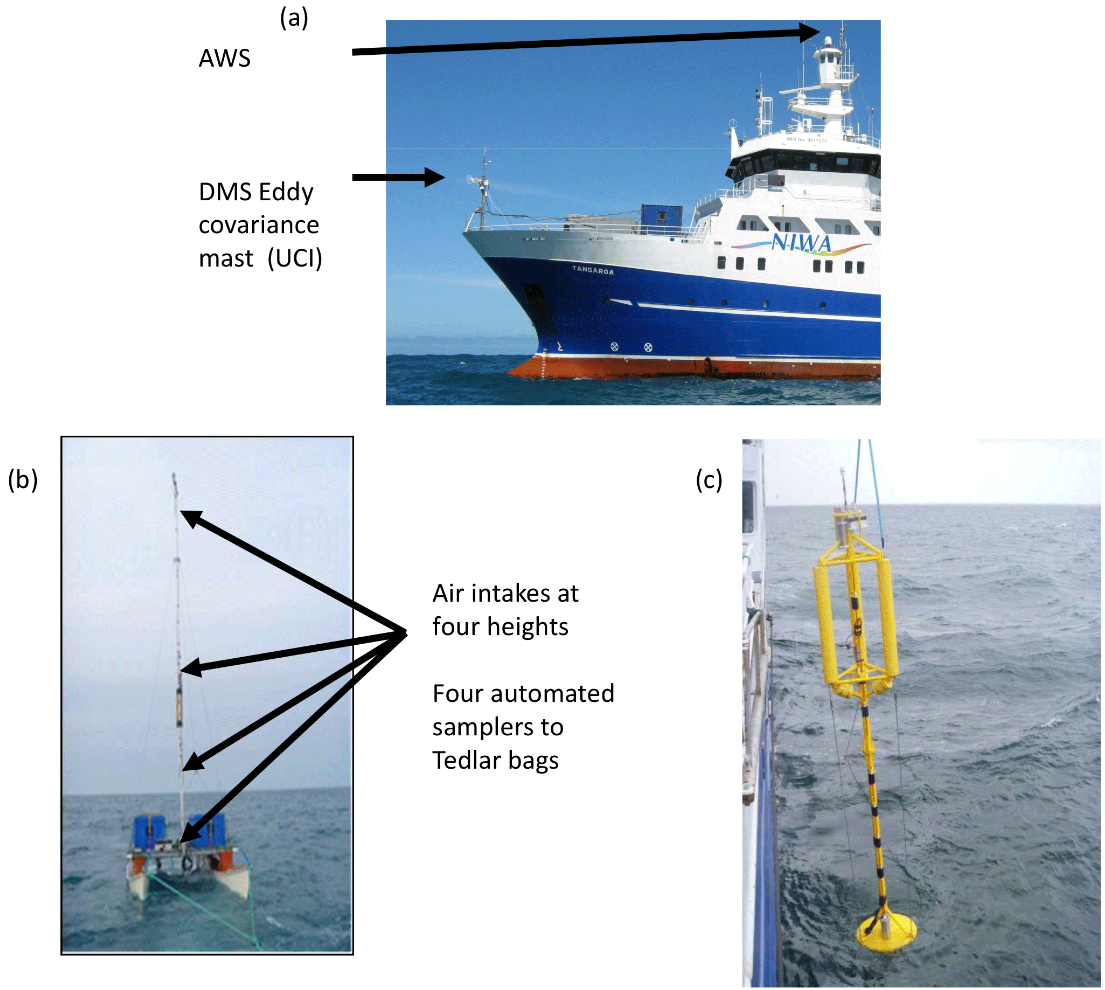
How to model the wind friction $\boldsymbol{\tau}$?

How is it linked to wave (and wave breaking) distributions ?



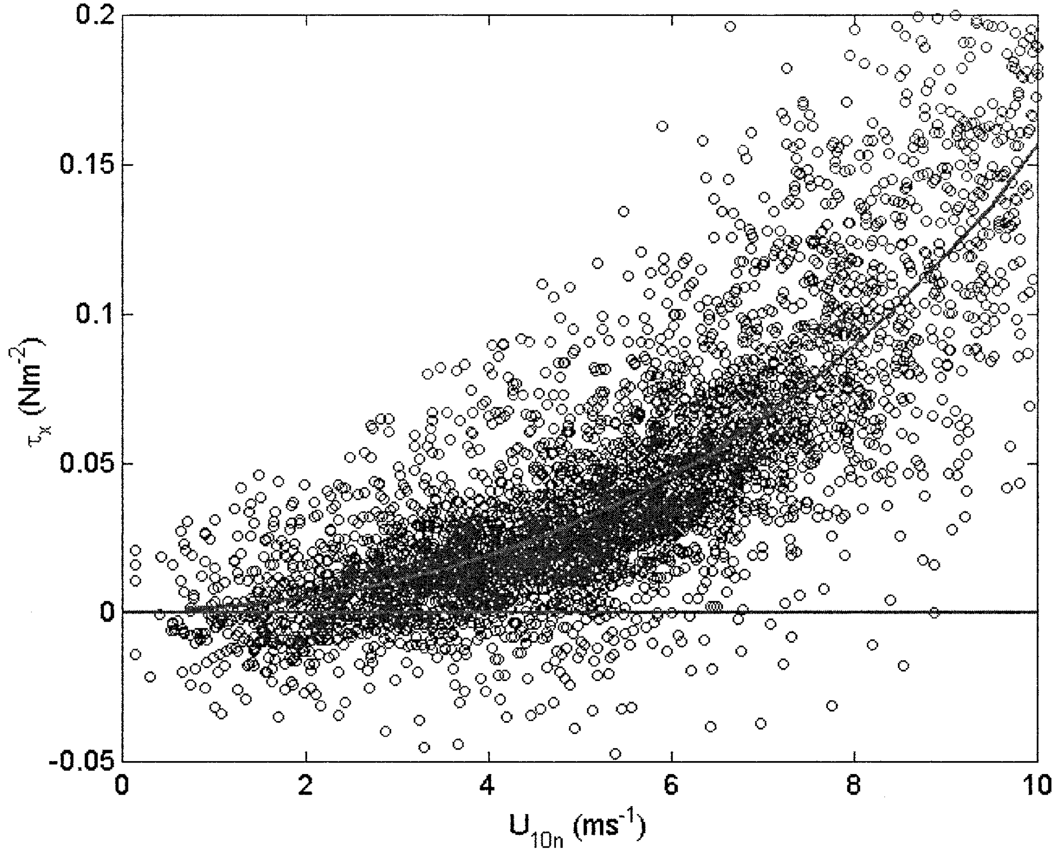
A simple model of energy injection (Jeffreys, 1922) : is it correct ?

$$S_{\rm in} \;=\; \frac{1}{2}\,\rho_a\, s_z\,(a\,k)^2\, c\,(U_z - c)^2$$

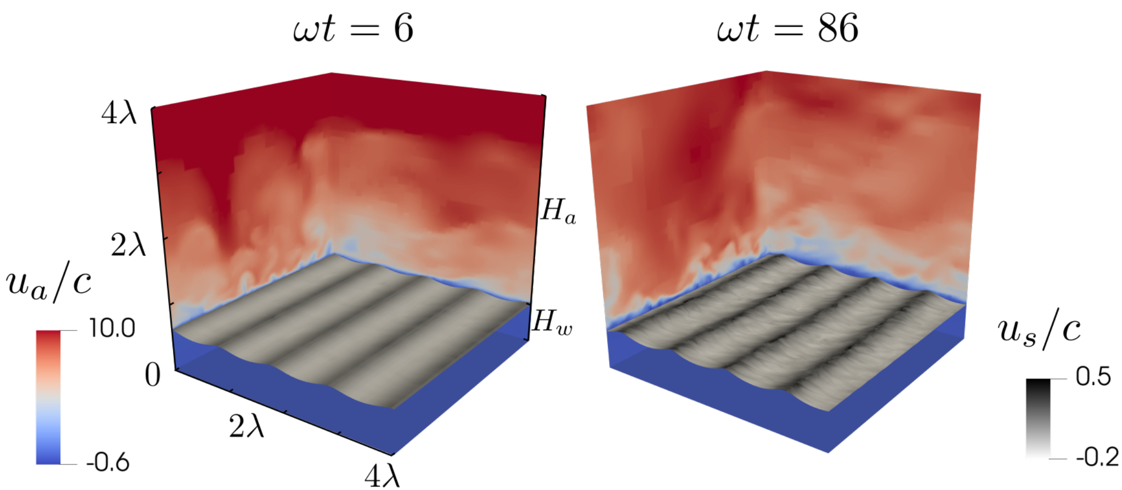


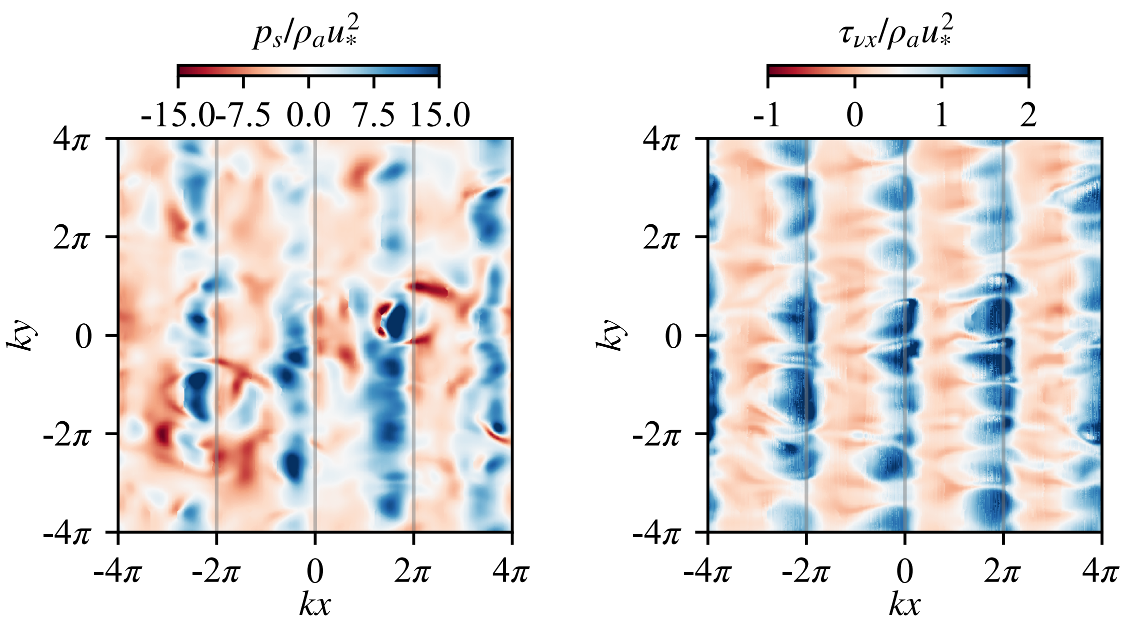
Aerosols/Eddy covariance measurements aboard R/V Tangaroa (Smith et al., Atmos. Chem. Phys. 2018).

Bulk flux (of heat, mass and momentum) parameterizations (Coupled Ocean–Atmosphere Response Experiment (COARE), Fairall et al, 2003).



Streamwise momentum flux as a function of wind speed from COARE
(Fairall et al, 2003, Journal of Climate)





$$\mathrm{Re}_\star = \frac{u_\star H_a}{\nu_a} = 720$$

$$\mathrm{Re}_w = \frac{c\,\lambda}{\nu_w} = 10^5$$

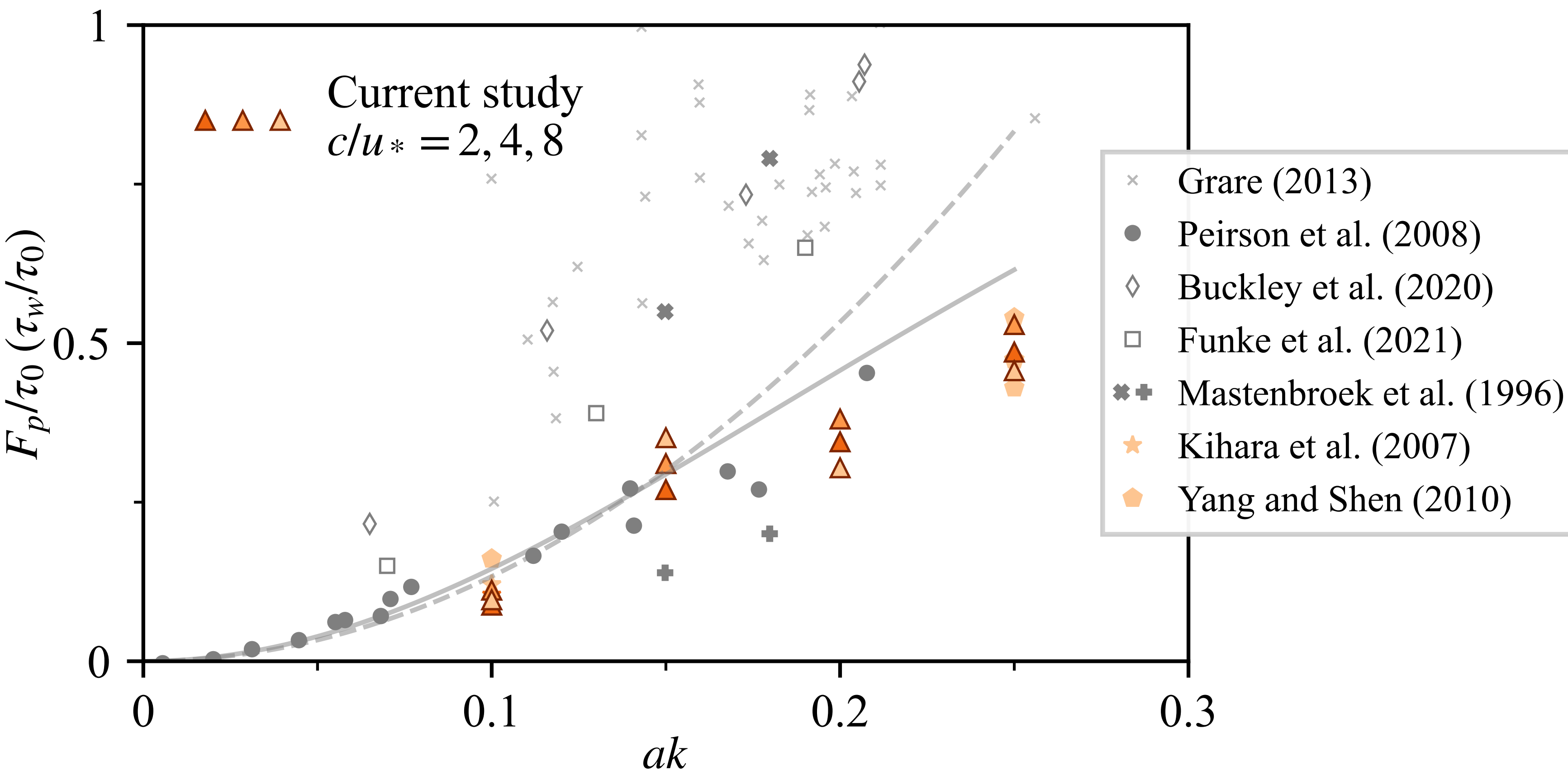
$$\mathrm{Bo} = 200$$

1024^3

Variable $c/u^\star$ and $a\,k$

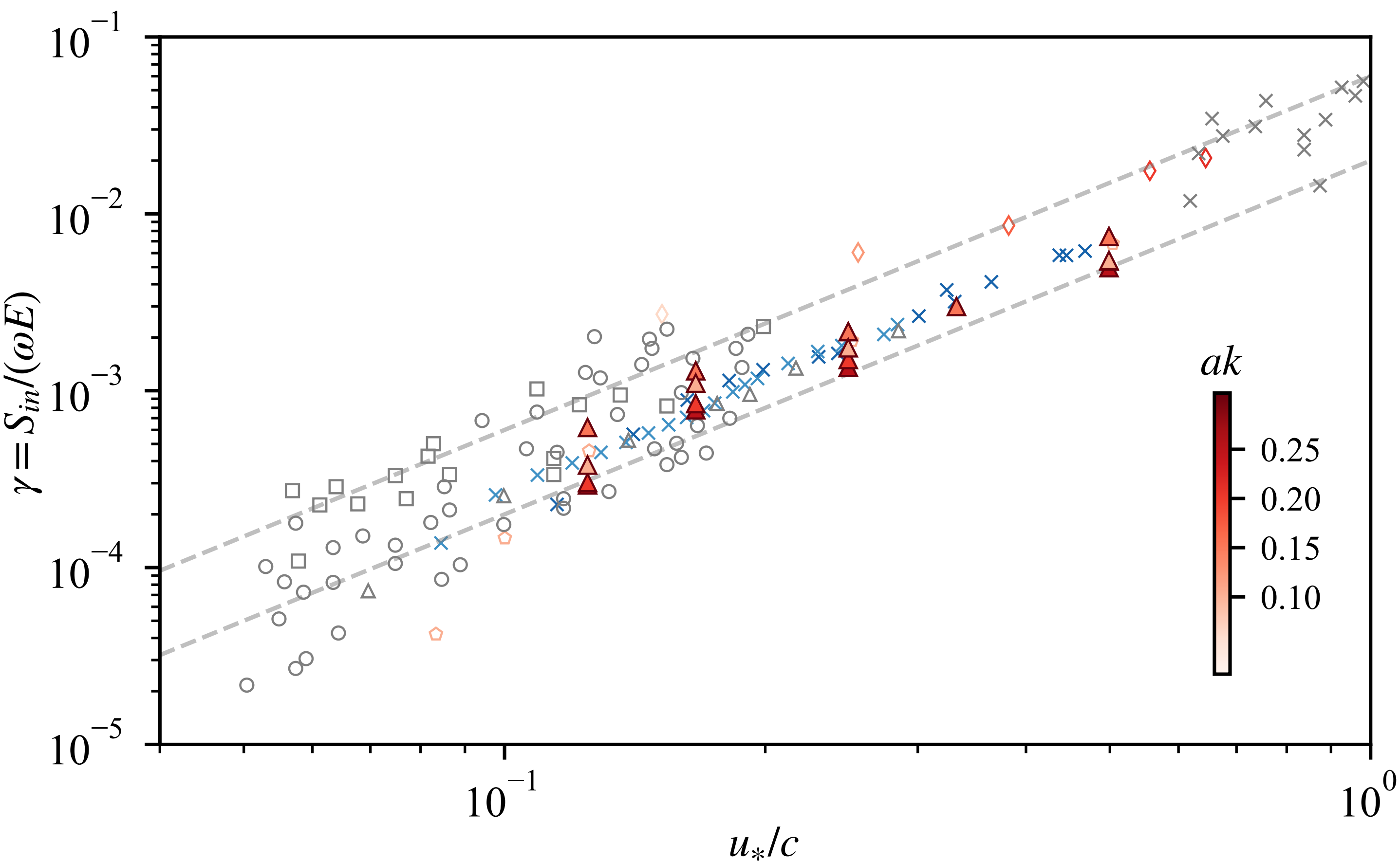
Comparison with lab experiments (not field data...) and other numerical results

$$S_{\text{in}} = \frac{1}{2}\rho_a\, s_z\, (a\,k)^2\, c\,(U_z - c)^2$$

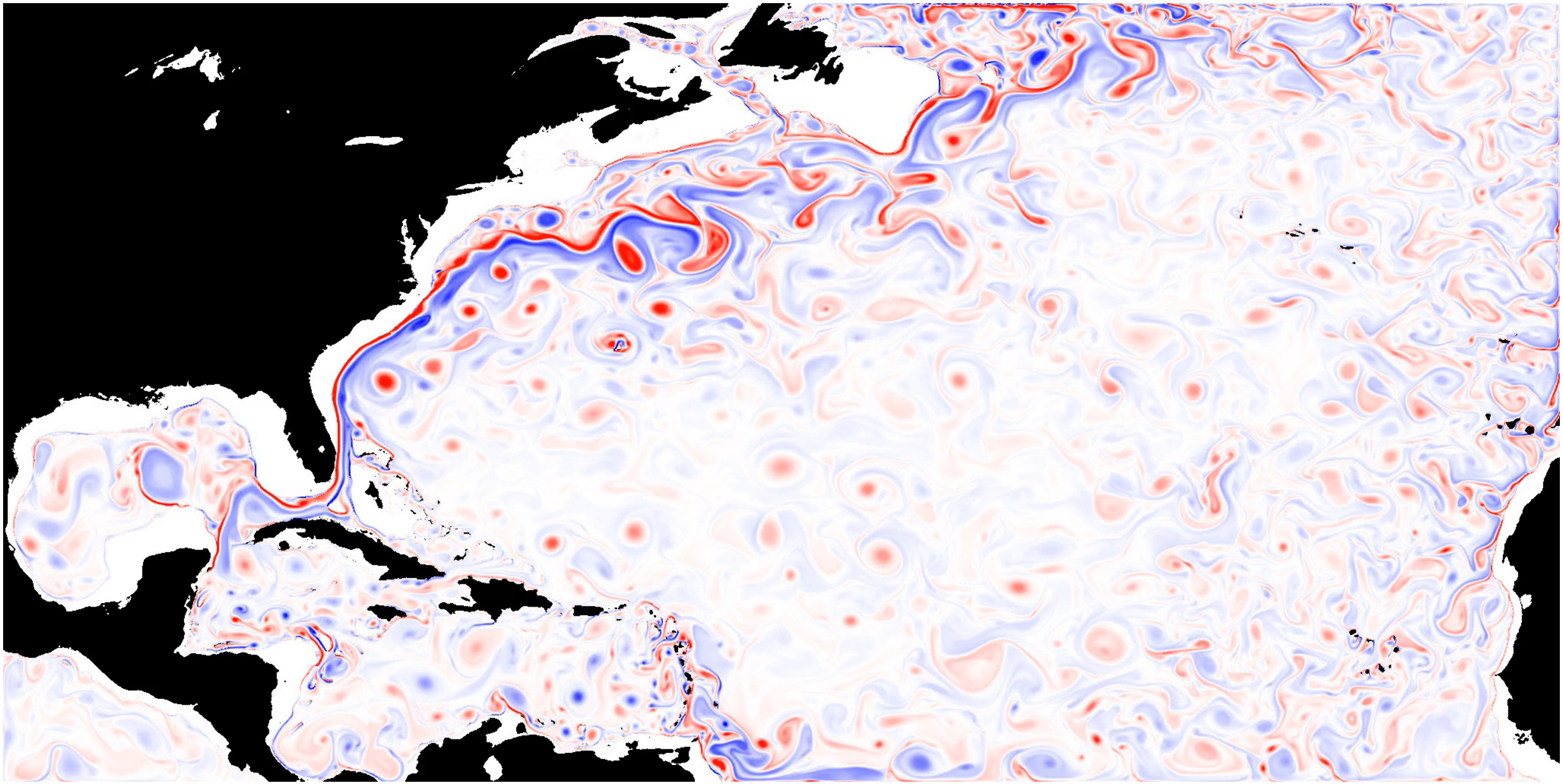


Comparison with lab experiments (Plant 1982) and a "spectral" numerical method (Yang et al 2013)

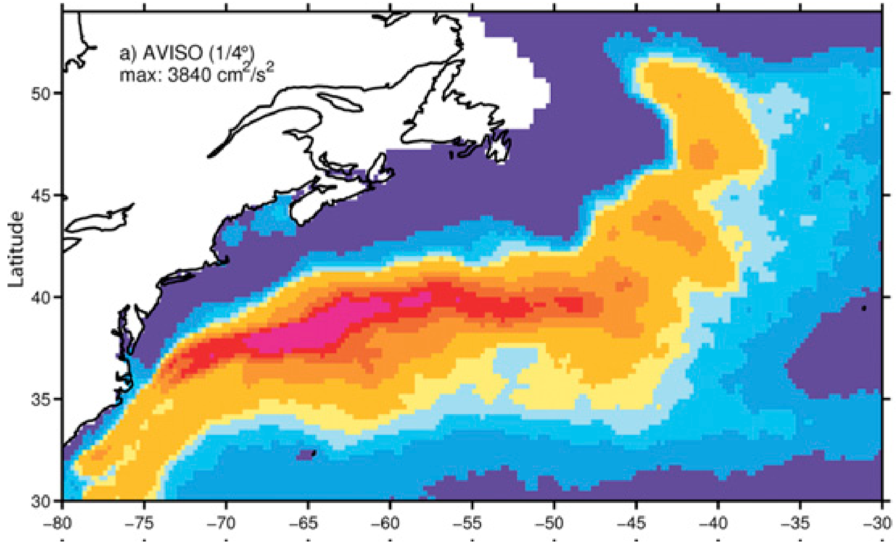
$$S_{\rm in} = \frac{1}{2}\rho_a\, s_z\,(a\,k)^2\, c\,(U_z - c)^2$$



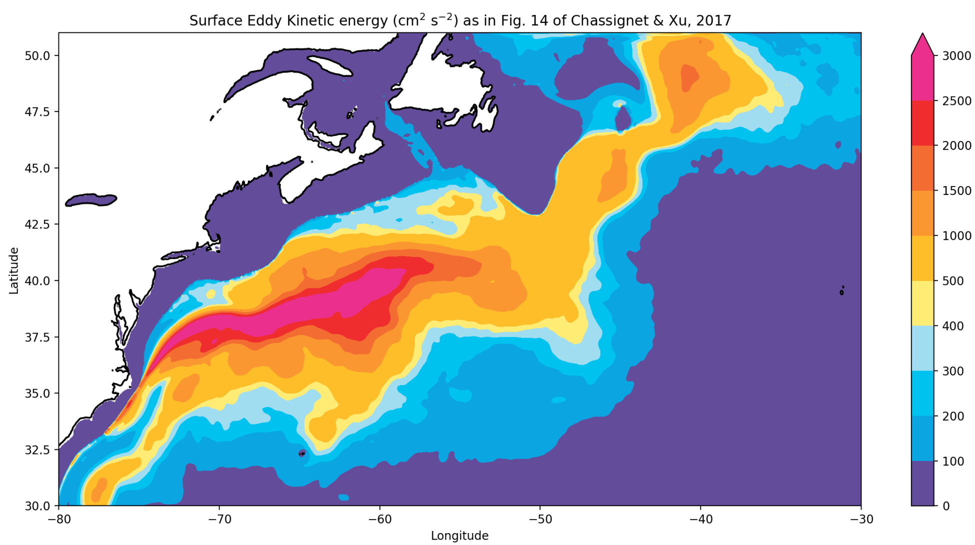
North Atlantic oceanic circulation simulated with the multilayer solver



Relative surface vorticity
Spatial resolution 1/24° (≈ 4.6 km), 5 layers, 23 years/day on 2048 cores
`basilisk.fr`

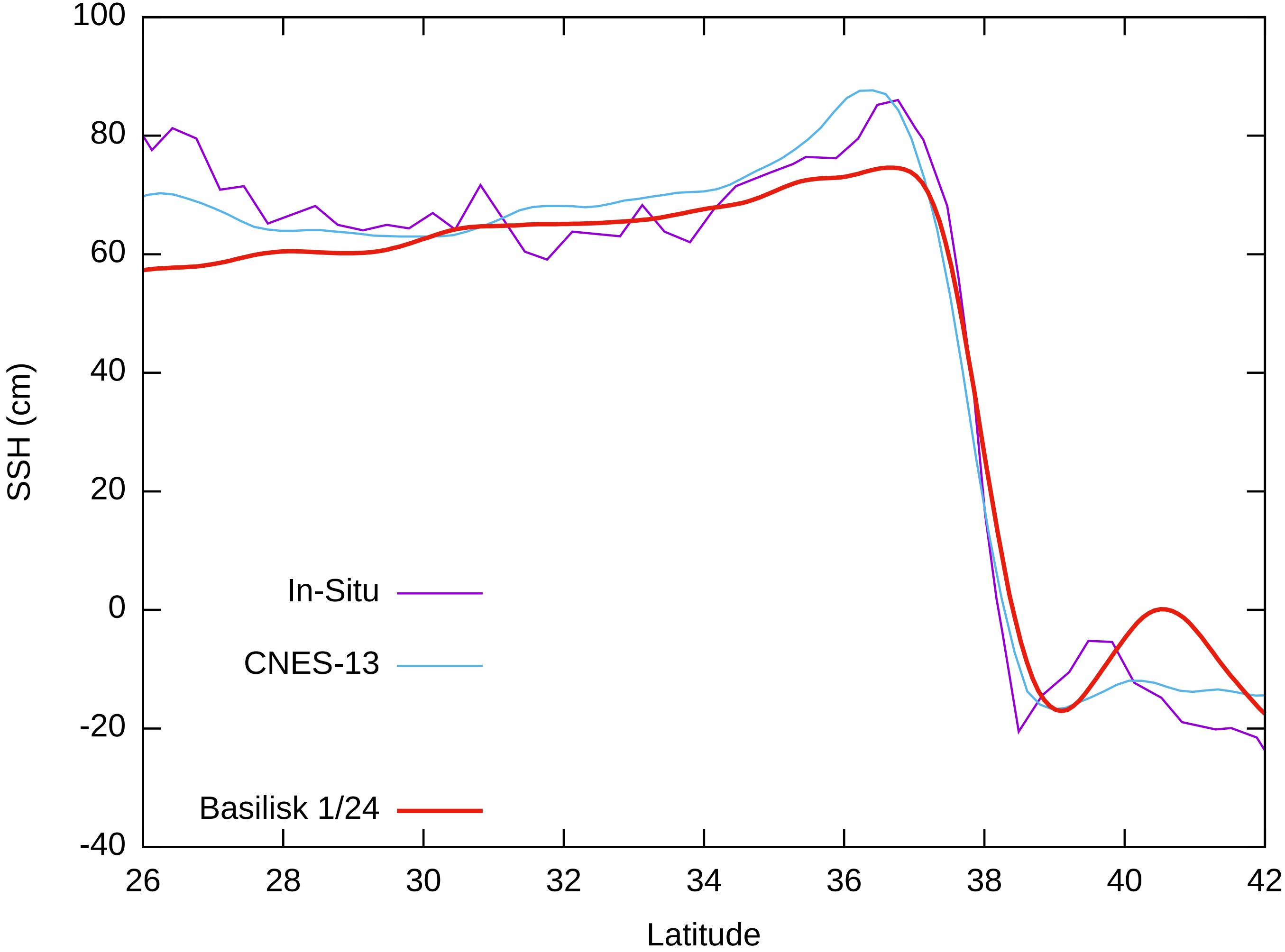


Aviso



Basilisk

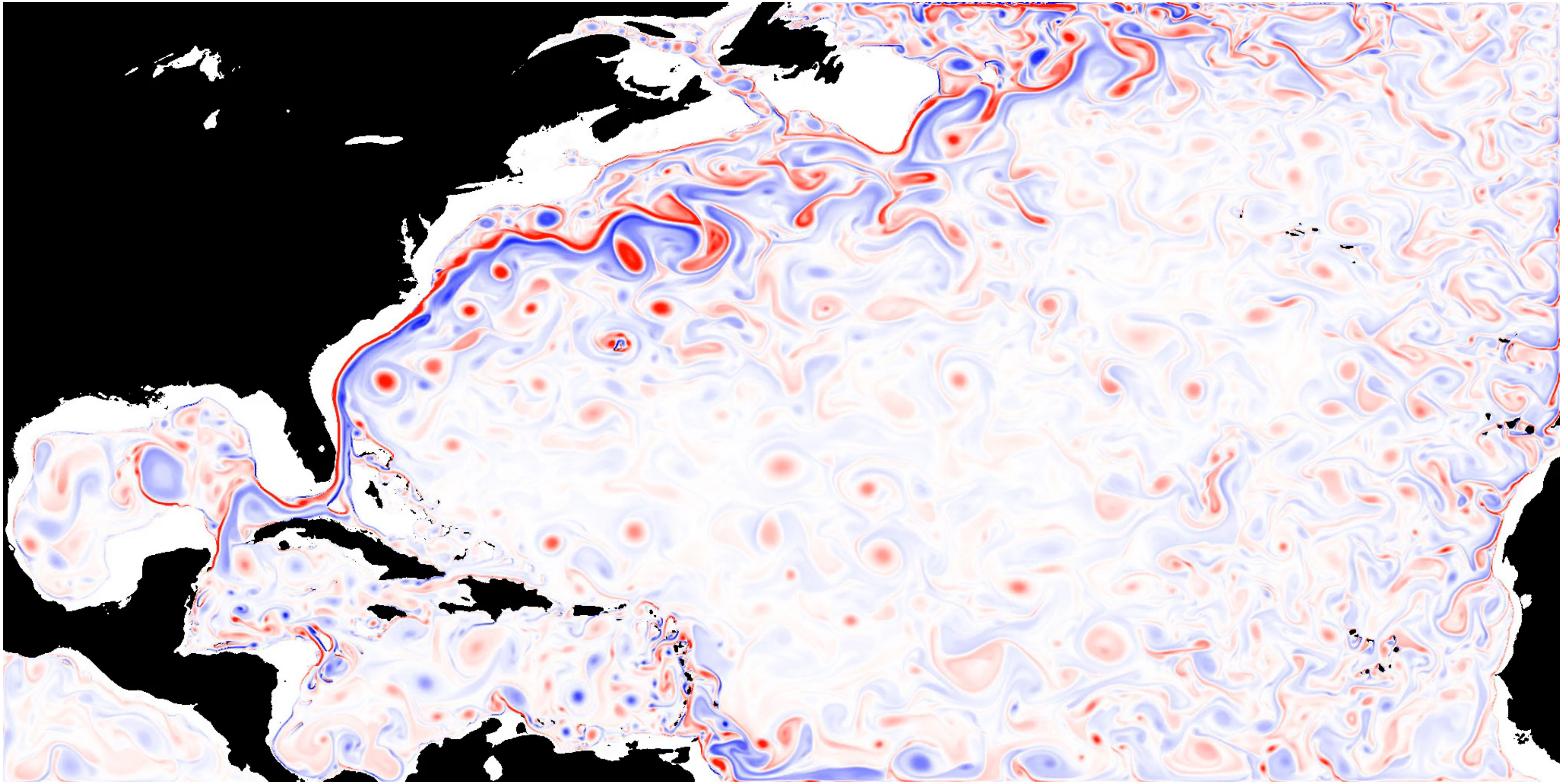
The Gulf Stream "step"



- We are trying to link microscale processes to the global scale
- Reduce the uncertainty and improve our understanding of the "climate-critical" ocean–atmosphere fluxes and other large-scale "parameterisations"
- This requires a broad range of fluid mechanics approaches (and collaborations)
- We use a combination of numerical approaches (several), simple physical models (e.g. for evaporation fluxes), statistical/dimensional analysis of (turbulent) processes
- It is important to start with the "classical" assumptions made in other fields (even when they have limitations...) and relate to well-known experimental/field datasets
- This is challenging and the road is long... (but we already have interesting results)
- "Scarce data" is (still) much more common in geophysics than "big data"

- Basilisk: open, collaborative and reproducible science

  `basilisk.fr`



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