Wave-structure interactions and wave energy

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GDR Climat

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Don't worry...

ferme houlomotrice pays basque

Images | Actualités | Vidéos | Livres | Finance

Environ 1740 résultats (0,24 secondes)

Ce serait le projet le plus avancé en France. Une ferme houlomotrice, capable de produire a minima 30 % de l'énergie des plus de 300.000 habitants du Pays basque en 2030 : c'est l'ambition affichée mercredi à Bayonne par la région Nouvelle-Aquitaine et la Communauté d'agglomération Pays basque.

13 juil. 2023

20 Minutes
https://www.20minutes.fr › planete › 4045468-202307...

Une ferme houlomotrice en projet au large de la ... - 20 Minutes

À propos des extraits optimisés • Commentaires

Communauté Pays Basque
https://www.communaute-paysbasque.fr › actualite › pro...

Projet de ferme houlomotrice au Pays Basque

13 juil. 2023 — Le site retenu pour accueillir la **ferme houlomotrice** est situé au large du phare de Biarritz, à une distance de 7,5 km de la côte et à une

Wave-Energy Convertors

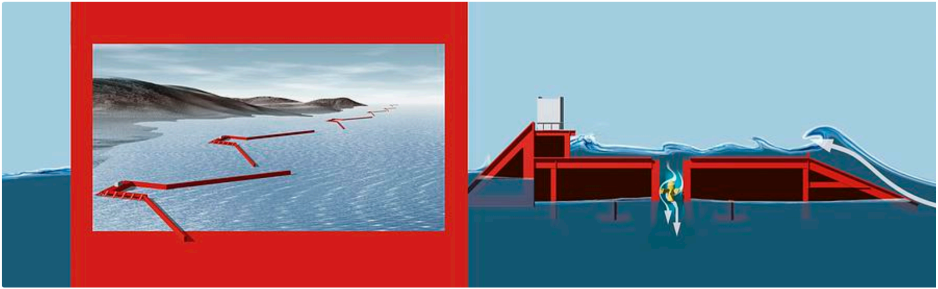

Diagram by Claus Lunau

Which energy ?

Free surface Euler equation

$$\text{(fluid domain)} \quad \begin{cases} \partial_t U + (U \cdot \nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z} P + \mathbf{g}, \\ \operatorname{div} U = 0, \end{cases}$$

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$$\text{(bottom)} \quad U \cdot N = 0$$

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Energy conservation

$$e = \frac{1}{2}\rho g \zeta^2 + \rho \int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

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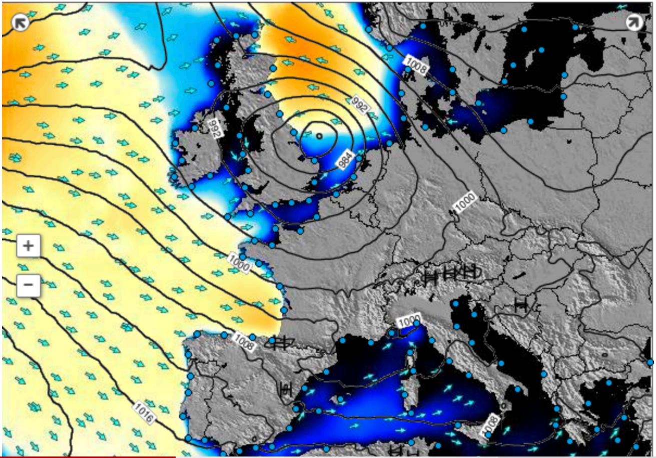
$$\mathcal{F} = \rho \int_{b(X)}^{\zeta(t,X)} V\big(g\zeta + \frac{1}{\rho}(P - P_{\mathrm{atm}}) + \frac{1}{2}|U|^2\big) \qquad (kW \cdot m^{-1})$$

Wave energy : how much ?

The annual average of wave energy in France, kilowatt/meter.

12.5 KW/m

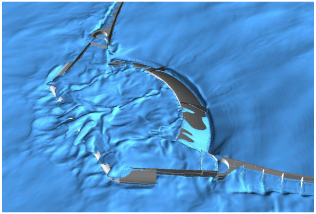


The situation today

Wave farms [edit]

Station	Country	Location	Capacity (MW)	Type	Operation	Notes
Ada Foah Wave Farm[1]	Ghana		0.4	Point absorber	2016	
Agucadoura Wave Farm (Pelamis).[2][3][4][5]	Portugal	41°25′57″N 08°50′33″W	2.25	Surface-following attenuator	July 2008-November 2008	
Azura[6]	United States		0.02	Point absorber	2015	
BOLT Lifesaver[7]	United States		0.03	Point absorber	2016	
CETO[8][9][10][11]	Australia	Western Australia			2015	Two submerged buoys anchored to the seabed generate energy through hydraulic pressure.
Gibraltar Wave Farm	Gibraltar	Gibraltar	.1	Surface attenuator	2016	
Islay Limpet[12][13]	United Kingdom	55°41′24″N 06°31′15″W	0.5	Oscillating water column	2000–2012	
Mutriku Breakwater Wave Plant[14][15][16]	Spain	43°18′26″N 2°23′6″W	0.3 (296 kW from 16 turbines and 16 OWCs.[17])	Oscillating water column	2011–date	Lifetime generation of over 3 GWh by the end of 2023.[18]
Ocean RusEnergy[19]	Russia	Yekaterinburg	N	Small-scale	2013	
Pico Wave Power Plant[20]	Portugal		0.4	Oscillating water column	2010	
[21]				Oscillating		

The CFD approach



[C. Eskilsson et al. 2015]

Specific difficulties ([Kim et al 2016])

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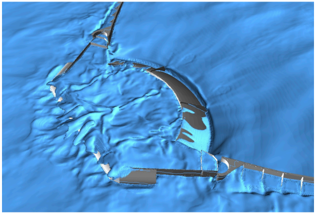

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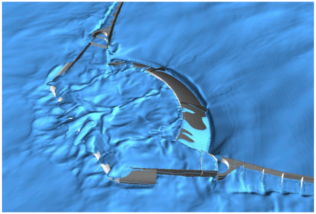

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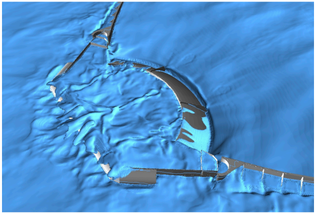

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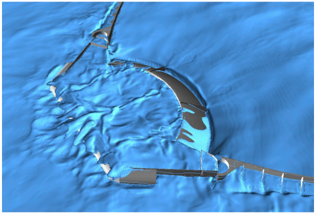

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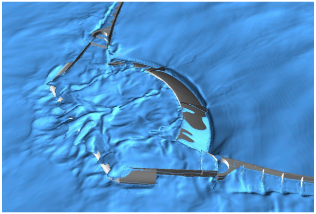

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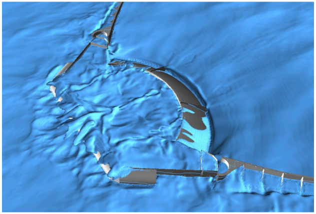

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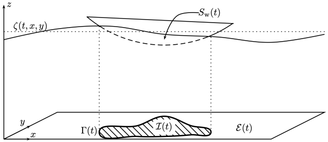
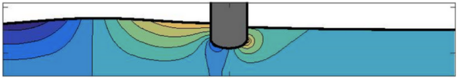
$\rightsquigarrow$ Cost of a CFD project for a numerical basin $\sim$ physical model tests.

$\rightsquigarrow$ Relevant for

1. Vortex-induced motion of a multi-column floater
2. Global performance of a multi-column floater in extreme wave

Fully nonlinear potential methods





[Engsig-Karup] [Dombre et al]

Free surface Bernoulli equation

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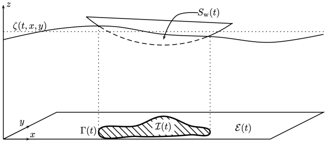
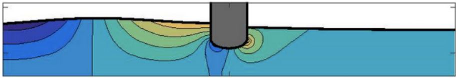
Fully nonlinear potential methods





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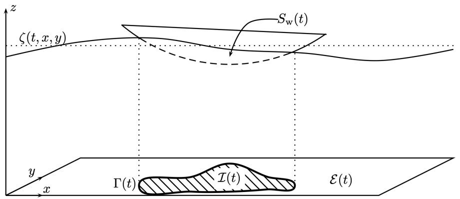
$\rightsquigarrow$ Faster than CFD but still very complicated

Linear potential methods



⇝ The variations of the fluid domain domain are neglected

$$\Phi = \dot{x}_j \psi_j + \int_{-\infty}^{t} \varphi_j(t-\tau)\dot{x}_j \mathrm{d}\tau$$

- ψ_j = potential for instantaneous impulsive velocity of floating object
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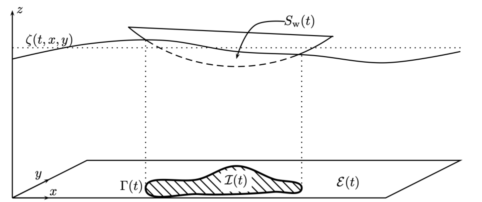

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⇝ Newton:

$$m\ddot{x} = m\mathbf{g} + \int_{\Gamma_\mathrm{w}} P\mathbf{n} \sim m\mathbf{g} + \int_{\Gamma_\mathrm{w}} (P_\mathrm{atm} - \rho g z - \partial_t \Phi)\mathbf{n}$$

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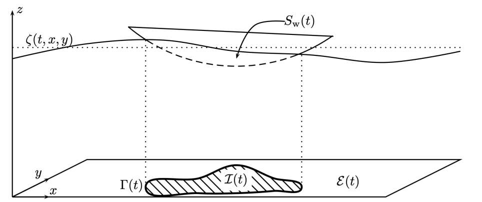

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⇝ Cummins' equation

$$\sum_{j=1}^{6} [(m_j\delta_{jk} + m_{jk})\ddot{x}_j + c_{jk}x_j + \int_{-\infty}^{t} K_{jk}(t-\tau)\dot{x}_j(\tau)\mathrm{d}\tau] = 0 \qquad (1 \le k \le 6)$$

Wave-Structure interaction as partially constrained models

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(i) *Choice of the wave model.* For instance, the NSW equations:

$$\begin{cases} \partial_t \zeta + \nabla \cdot (hv) = 0, \\ \partial_t v + v \cdot \nabla v + g\nabla\zeta = -\frac{1}{\rho}\nabla \underline{P}, \end{cases} \qquad (h = H_0 + \zeta)$$

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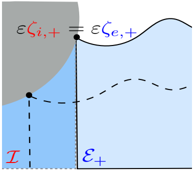
(iii) *Matching conditions on Γ.* What ?

The contacts points...

- Non-vertical walls
  - continuity of q
  - continuity of ζ
  - continuity of $\underline{P}$

$\rightsquigarrow$ **free boundary problem**



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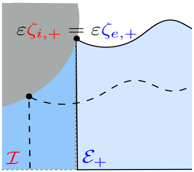

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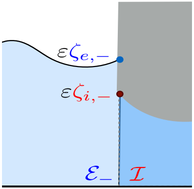

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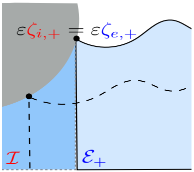

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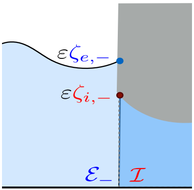
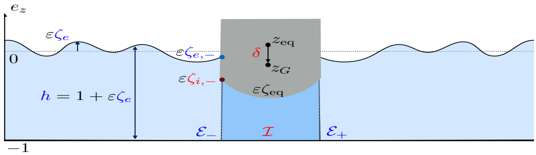


$\rightsquigarrow$ Continuity of the energy flux

Reduction to an IBVP/ transmission problem



Notations:

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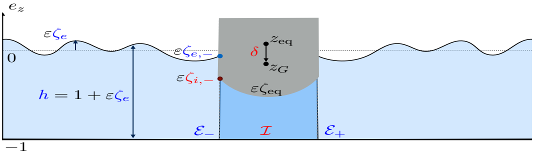

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- Exterior

$$\begin{cases} \partial_t \zeta_e + \partial_x q_e = 0 \\ \partial_t q_e + \partial_x \left(\frac{{q_e}^2}{h_e}\right) + g\, h_e \partial_x \zeta_e = 0 \end{cases}$$

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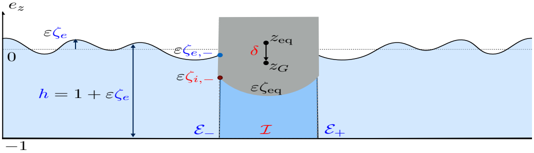

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where

$$\begin{cases} (m + \mathfrak{m}(\delta))\ddot{\delta} + \delta + \mathrm{NL}(\delta, \langle q_i \rangle) & = \langle \mathfrak{G}_e \rangle \\ \alpha(\varepsilon\delta)\frac{d}{dt}\langle q_i \rangle + \mathrm{NL}(\delta, \langle q_i \rangle) & = -\frac{[\![\mathfrak{G}_e]\!]}{2\lambda} \end{cases}$$

$$\mathfrak{G} := g\zeta + \frac{1}{2}\frac{q^2}{h^2}$$

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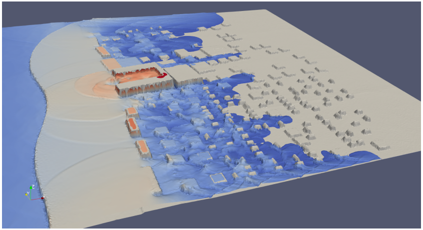
1. CFD: very precise, very CPU expensive
2. Fully nonlinear potential flow: precise, CPU expensive
3. Constrained reduced models
   - Limited range of validity
   - Very fast and capture nonlinear effects
   - Can be coupled to existing operational wave models



Uhaina

4. Linear potential flow: low precision, very fast

Conclusion

Various strategies:

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2. Fully nonlinear potential flow: precise, CPU expensive
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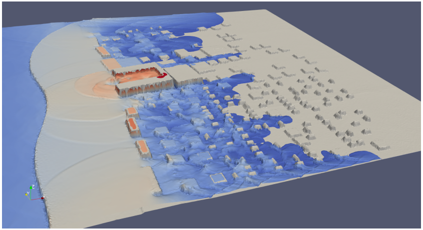

Uhaina

   - Open mathematical and numerical issues for the treatment of IBVP

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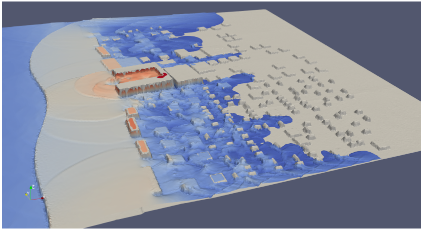

Uhaina

   - Open mathematical and numerical issues for the treatment of IBVP
   - Extension to see-ice modelling ?
4. Linear potential flow: low precision, very fast

Thank you for your attention!